Close Tue: 10.2/13.2, 10.3
Close Thu: 13.3 (finish much sooner)
Midterm 1, Thursday, Apr. 20th
Covers 12.1-12.5, 10.1-10.3, 13.1-13.3

Today: A bit of 10.2/13.2 (calculus on curves), then 10.3 (polar coordinates)

10.2/13.2 Calculus on curves

This first page is review from Math 124 (read 10.2 for a refresher).
Going from 2D parametric to slope and concavity:

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad \text{and} \quad \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(f'(x)\right)}{dx/dt}$$

Entry Task: Consider

$$x = t, y = 2 - t^2$$

(a) Find dy/dx and d^2y/dx^2.

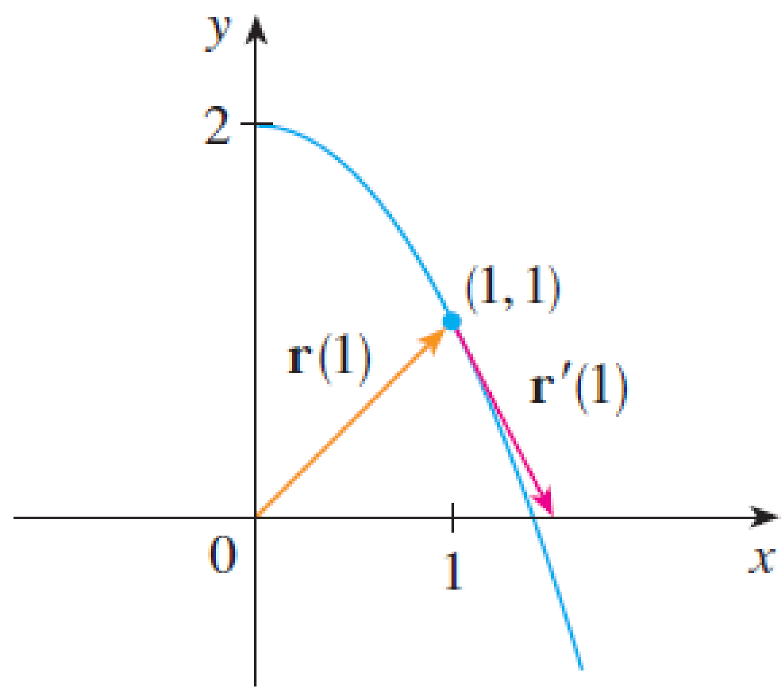
(b) Find the equation for the tangent line at t = 3.
(put in form $y = mx + b$).

New: Consider

$$\boldsymbol{r}(t) = \langle t, 2 - t^2 \rangle$$

(a) Find a tangent vector.

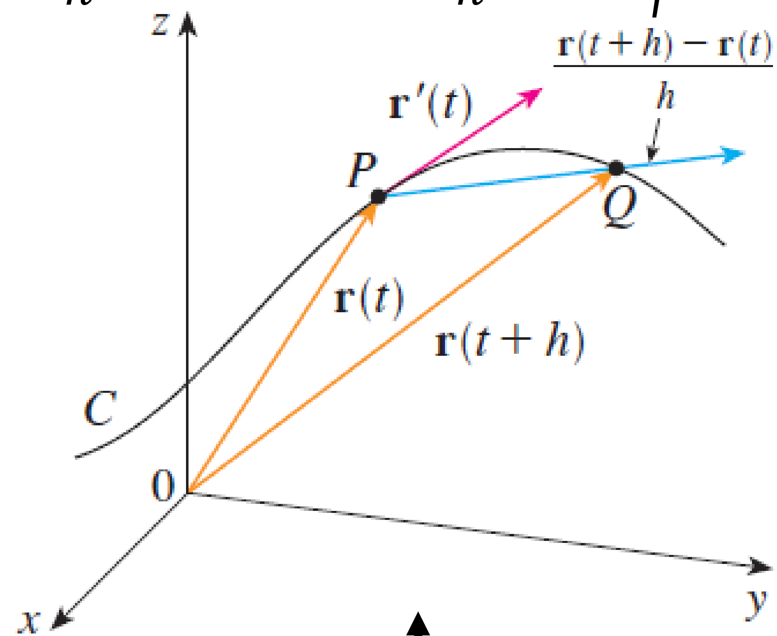
(b) Find parametric equations for the tangent line at t = 3.



In general: (Vector Calculus)

We define $\vec{\boldsymbol{r}}'(t) = \lim_{h\to 0}\left\langle \frac{x(t+h)-x(t)}{h}, \frac{y(t+h)-y(t)}{h}, \frac{z(t+h)-z(t)}{h} \right\rangle$

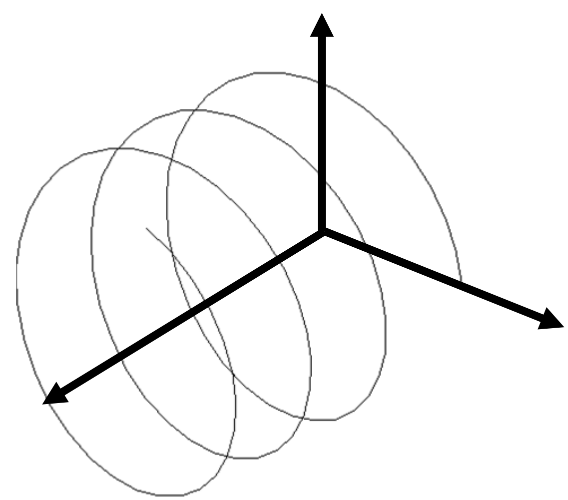
Thus, $\vec{\boldsymbol{r}}'(t) = \langle x'(t), y'(t), z'(t)\rangle$.
Morale, do calculus **component-wise**.



Example: $\vec{\boldsymbol{r}}(t) = \langle t, \cos(2t), \sin(2t)\rangle$.

(a) Find $\vec{\boldsymbol{r}}'(t)$.

(b) Give parametric equations for the tangent line at $t = \pi/4$.



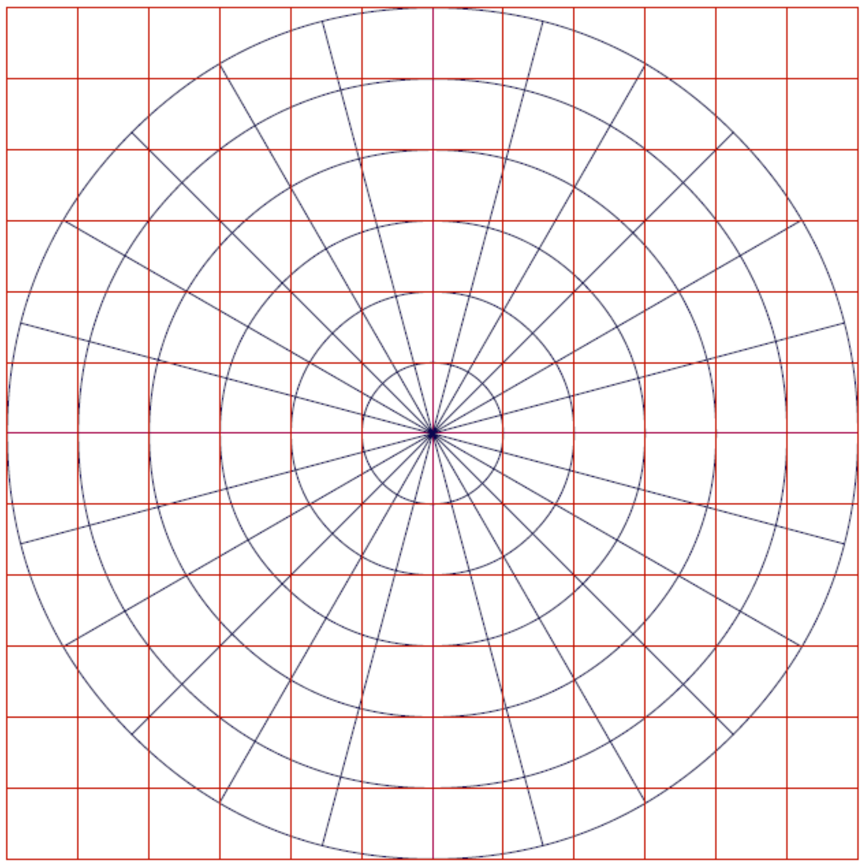
10.3 Polar Coordinates

Goal: A 2D coordinate system good for describing circular/arcing paths.

Cartesian	**Polar**
Given (x, y) 1. Stand at origin.	Given (r, θ) 1. Stand at origin facing the positive *x*-axis.
2. Move x-units on x-axis. pos. = right, neg. = left	2. Rotate by θ. pos. = ccw, neg. = clockwise
3. Move y-units parallel to y-axis. pos. = up neg. = down	3. Walk *r*-units in direction you are facing. pos. = forward neg. = backward

Example: Plot these polar points

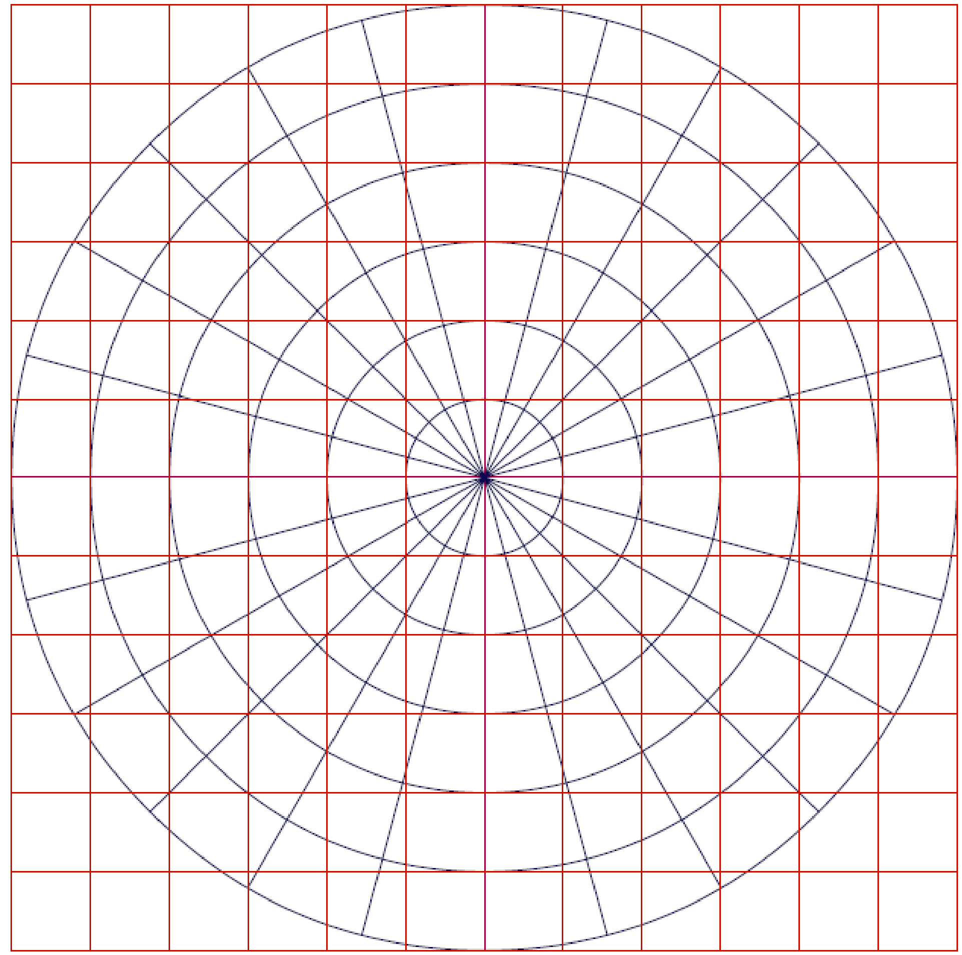
1. $(r, \theta) = (1, \pi/2)$
2. $(r, \theta) = (3, 5\pi/4)$
3. $(r, \theta) = (0, \pi/3)$
4. $(r, \theta) = (-1, 3\pi/2)$
5. $(r, \theta) = (4, 0)$
6. $(r, \theta) = (4, 100\pi)$



From trig we already know:

$$x = r\cos(\theta), \qquad y = r\sin(\theta)$$

$$\tan(\theta) = \frac{y}{x}, \qquad x^2 + y^2 = r^2$$



Exercise:

1. Describe all pts where *r* = 3.

2. Describe all pts where θ = π/4.

Plotting Polar Curves

Option 1: Try to convert to x and y.
Then hope you recognize the curve.

Option 2: **Plot points!**
Start with 0, $\pi/2$, π, $3\pi/2$. For more detail do multiples of $\pi/6$ and $\pi/4$.

Option 3: Do some calculus first.

If $r = f(\theta)$, then

$$x = rcos(\theta) = f(\theta)\cos(\theta)$$
$$y = rsin(\theta) = f(\theta)\sin(\theta)$$

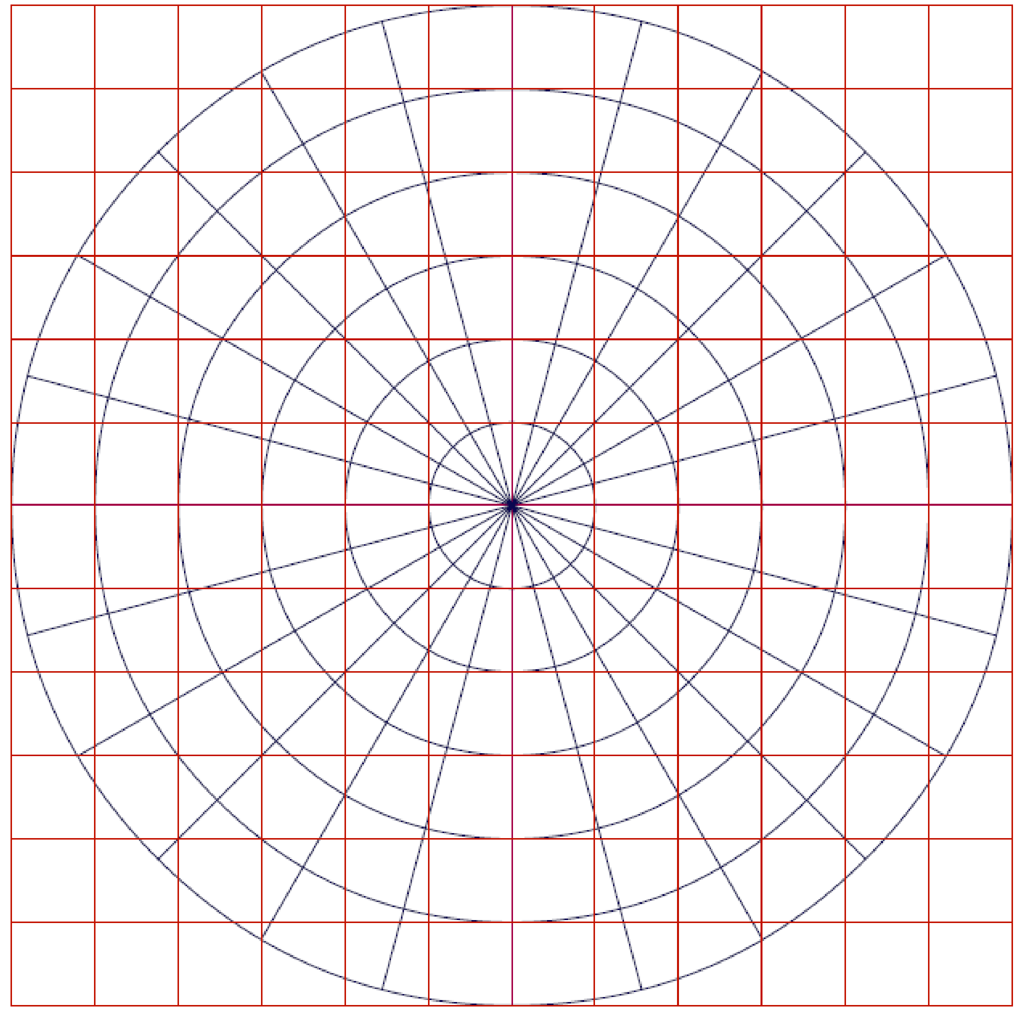
so

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$
$$= \frac{f'(\theta)\sin(\theta) + f(\theta)\cos(\theta)}{f'(\theta)\cos(\theta) - f(\theta)\sin(\theta)}$$

Example: Graph r = sin(θ)

θ	0	π/2	π	3π/2	2π
r					

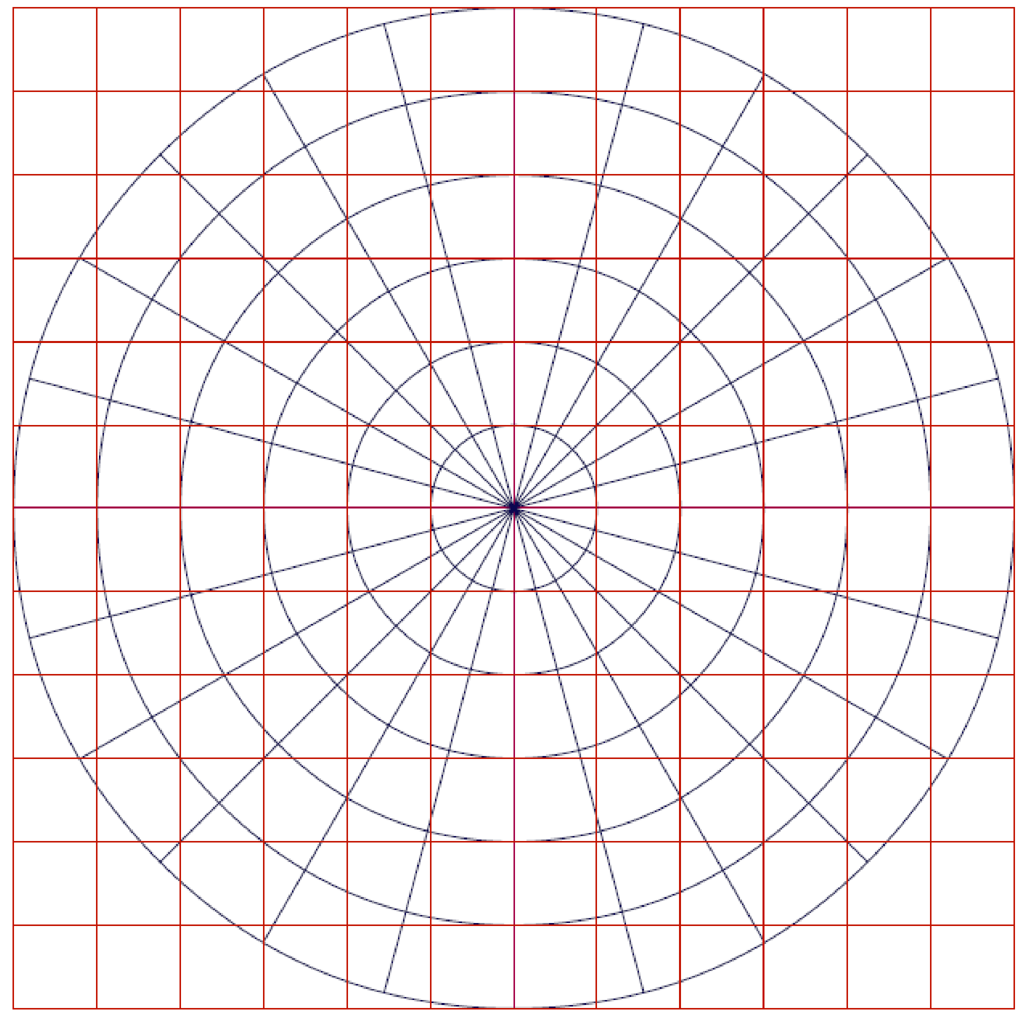
θ	π/6	π/4	π/3	2π/3	3π/4	5π/6
r						



Example: Graph r = cos(2θ)

θ	0	π/2	π	3π/2	2π
r					

θ	π/6	π/4	π/3	2π/3	3π/4	5π/6
r						



An old exam question:

The four polar equations below each match up with one of the six pictures. Identify which match.

1. $r = \sqrt{\theta}$
2. $r = 1 - 2\cos(\theta)$
3. $r = 1 + \sin(2\theta)$
4. $r = 9\cos(\theta)$